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Constraints on theories with large extra dimensions

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Abstract: Recently, a number of authors have challenged the conventional assumption that the string scale, Planck mass, and unification scale are roughly comparable. It has been suggested that the string scale could be as low as a TeV. In this note, we explore constraints on these scenarios. We argue that the most plausible cases have a fundamental scale of at least 10 TeV and five dimensions of inverse size 10 MeV. We show that a radial dilaton mass in the range of proposed millimeter scale gravitational experiments arises naturally in these scenarios. Most other scenarios require huge values of flux and may not be realizable in M Theory. Existing precision experiments put a conservative lower bound of $6-10$ TeV on the fundamental energy scale. We note that large dimensions with bulk supersymmetry might be a natural framework for quintessence, and make some other tentative remarks about cosmology.

Keywords: Superstrings and Heterotic Strings, Superstring Vacua.

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Contents

1. Introduction

It has been traditional, in string theory, to assume that the compactification scale and the string scale are both large, comparable to the Planck scale. This stemmed from the belief that sensible string phenomenology could only emerge from the heterotic string. In weakly coupled heterotic string theory, the coincidence of these scales is inevitable, since the observed four dimensional gauge couplings are inversely proportional to the volume of the compact space, V_d. With the recent discovery that all string theories are different limits of a larger theory, called M Theory, these assumptions have been called into question. Indeed, they do not even hold in weakly coupled Type I theory. Witten argued that the phenomenon of coupling constant unification suggests that the proper description of nature might be as a strongly coupled heterotic string, in which the fundamental Planck scale was of order the unification scale, with the eleventh dimension about 40 times larger [1]. More generally, it has been realized that in M Theory, nonabelian gauge fields arise on lower dimensional submanifolds of the internal space, which have been named *branes*. It is thus quite natural to have an internal manifold somewhat larger than the fundamental scale, which would then give a large effective four dimensional Planck mass, without affecting the gauge couplings.

Ref. [2] pointed out a difficulty with this sort of picture. In a theory where at least five dimensions are large, it is difficult to understand the stabilization of the radii. If the underlying theory is supersymmetric, the bulk supersymmetry, which is at least that of a five dimensional theory, requires that the potential vanish for large radii. This means that the radii must be stabilized at values of order the fundamental scale. This is similar to the argument that the gauge coupling must be of order one. In the case of the gauge coupling, the problem is to explain why the coupling is 1/25 instead of one, and why a weak coupling expansion should hold in the low energy theory. For the case of the scales of a supersymmetric theory, one must understand why there are ratios of scales of order 40 rather than 1, and why a large radius M-theory type picture is valid. A possible explanation why numbers normally of order one might be $10-100$ was described in [3] and [2]. Note also, that in Witten's picture there is only one large dimension, whereas in general there might be as many as seven. This would further reduce the size of the internal manifold necessary to the explanation of the discrepancy between the unification and Planck scales.

While explaining numbers of order $10-100$ may be troubling, the only enormous hierarchies in these pictures which require explanation are the ratio of M_w over M_p, and the smallness of the cosmological constant. The first problem is traditionally explained through exponentially small low energy field theory effects, and the latter problem is a feature of any description of the world in terms of low energy effective field theory.[1]

By contrast, the radii, in units of the fundamental Planck scale or string tension, are at most numbers of order a few (perhaps 70 in the Witten proposal, and smaller in generalizations of it with more dimensions a bit bigger than the GUT scale), and the basic couplings are numbers of order one. The coincidence of the radial size with the GUT scale allows us to understand the evidence for coupling unification and neutrino masses as probes of the fundamental scale of the theory, while the Planck scale arises from the product of a large number of numbers of order one, and has no fundamental significance.[2] The high fundamental scale also makes it easier to suppress violations of baryon number, lepton number and flavor, though in a theory with standard model superpartners around the TeV scale, one also needs discrete symmetries to ensure that these approximate conservation laws are preserved with the requisite accuracy.

More recently, a number of authors have considered the possibility that the compactification (energy) scale is far lower, with the fundamental scale of string theory being as low as a TeV [6, 7, 8, 9, 10]. They argue, first, that this might be desirable. Scalar masses would then naturally be of order 1 TeV or so, eliminating the conventional fine tuning problem associated with the Higgs boson. Proton decay could be forbidden by discrete symmetries, much as in the more conventional supersymmetric case, though more intricate symmetries may be required (bulk gauge symmetries were suggested in [11]). Flavor problems can be suppressed if one supposes that

[1] As it is conventionally understood. See however [4, 5].

[2] We note also that the simplest way to understand the magnitude of density fluctuations as measured by COBE, invokes an inflationary potential of the form $M_{GUT}^4 v(\phi/M_P)$. This suggests again that M_{GUT} is the fundamental scale and that the inflaton is a bulk modulus in a brane scenario with SUSY broken only on branes.

there are sufficiently elaborate non-abelian flavor symmetries. More serious issues are posed by the production of the light Kaluza-Klein (KK) states. But these authors show that provided the compactification radius is smaller than a millimeter, then production of these particles is not a serious problem [12]. Accelerator experiments set limits in the TeV range [13], while astrophysics in some cases sets stronger limits [12]. In particular, in the scenario with two large dimensions, where KK states might be observable in sensitive gravitational experiments, the bounds on the higher dimensional Planck scale are pushed up to 30 TeV.

The possibility that we might observe violations of Newton's law of gravity in sensitive tabletop experiments, or probe extra dimensions in accelerators is extremely exciting. The fact that such a possibility is not a priori ridiculous is very interesting. But it is natural to ask: how plausible is it that the extra dimensions be so large?

In this note, we will address this question from several points of view. We should stress from the start that we will not be able to demonstrate that a low string scale and large compact dimensions are impossible. We will concentrate on scenarios in which the fundamental scale is within the reach of planned accelerator experiments. Some of our remarks and techniques may be applicable to the analysis of other parts of the wide spectrum of choices which have been explored for the sizes of extra dimensions [14] but we will not discuss these more general scenarios. Our discussion of laboratory constraints is definitely not relevant to most of these more general scenarios.

Our discussion will focus on three issues:

- The problem of fixing the moduli: This problem takes on different forms according to the manner in which SUSY is broken. There are two ways to break SUSY consistent with observation. It can be broken (at or near the fundamental TeV scale) on the brane where the standard model lives, but unbroken in the bulk. Or it can be broken everywhere at the fundamental scale. In the second case, there is an essentially unique way to stabilize the radial dilaton, which is to balance the energy of a large flux against the bulk cosmological constant, as proposed by Arkani-Hamed, Dimopoulos and March-Russell [15], following earlier suggestions by Sundrum [16]. (Other discussions of stabilization appear in [17].) We show that in this case, the requisite fluxes are huge, always larger than 10^5. When SUSY is (approximately) preserved in the bulk, the mechanisms of stabilization are more intricate. In many cases the radial dilaton mass is extremely small, and it is likely to lead to observational problems. However, there is a scenario with 5 relatively large dimensions and a fundamental scale of order 10 TeV, where the flux needed to stabilize the radius is of order one.[3]

[3]We must also invoke a plausible weak coupling factor and a scale of SUSY breaking of order 1 TeV on the brane.

In this case the radial dilaton is within range of proposed improvements of the Cavendish experiment.

We emphasize however that in most of these models, the hierarchy problem is solved in the low energy effective theory by introducing a large integer valued conserved flux on a compact manifold. There is no guarantee that M Theory allows such large fluxes, and there are no known supersymmetric vacua with flat compact dimensions which support such large values of the flux.

Finally, we note that if we do not stabilize the radial dilaton in the SUSY case, we might be led to an interesting model of quintessence [18, 19, 20, 21, 22] This possibility is potentially exciting for several reasons. First, it might provide a natural understanding of the large value of the radius. In addition, the scale of supersymmetry breaking and of the vacuum energy are not so closely tied as in more conventional pictures. As we will explain, quintessence based on bulk moduli fields in a brane world does not have a problem with the time variation of the fine structure constant. Unfortunately, bounds on the time variation of Newton's constant require that the radial dilaton did not change drastically during post BBN cosmological history (so that a cosmological explanation for the large size of the extra dimensions must involve cosmology before this time). In addition, the radial dilaton is extremely light, and generically possesses Brans-Dicke couplings of order one, which will cause problems for astronomy.

- Problems of flavor: Here we include not just flavor, but also the effects of higher dimension flavor-conserving operators. These suggest lower limits of order 6 TeV on the scale of the theory. As explained in ref. [11, 23], one can hypothesize mechanisms for fermion mass generation which lead to suitable textures, and these mechanisms can suppress many dangerous flavor-changing processes. There are two difficulties, however, one theoretical and one phenomenological. The flavor proposals of [11, 23] require that flavor symmetries be broken on "other, faraway" branes from the one on which the standard model fields live. But, as we will explain, given the lack of supersymmetry one expects that there is a potential between the branes, and they are not likely to be widely separated. This flaw, as we will see, is hardly fatal, but it is still one more coincidence required for the whole picture to make sense. Second, these mechanisms do not account by themselves for the smallness of CP violation. Without supposing additional structure or very weak couplings, one obtains a limit of at least 10 TeV. These scales are already troubling from the point of view of scalar masses. One needs to explain why the Higgsmass-squared is 100 to 1000 times smaller than its expected value. This criticism is certainly not decisive, and it also applies to many supersymmetric models.

- Cosmology. Here our statements will be quite tentative. We know little about the cosmology of such high dimension theories. Still, there are several puzzles. Most of these are connected with the question of how the theory finds its way into the correct vacuum. Essentially all of them lead back to an inflationary era [24, 27, 28, 26]. Inflationary cosmology in a brane world scenario has not been sufficiently explored to warrant definite conclusions. We will simply exhibit the formidable problems to be solved and make a few remarks about attempts to solve them. In particular, in arguing that scenarios with millimeter extra dimensions were compatible with Big Bang Nucleosynthesis, the authors of [12] assume initial conditions in which the brane is excited to about 1 MeV, while bulk modes of much lower energy are in their ground state. We suggest that it may be very difficult to find plausible conditions at higher energy which leave the system in this rather bizarre state. In particular, although Dvali and Tye [29] have proposed an explicit model in which a field on the brane dominates the energy density at some point in time, the reheat temperature they obtain is too high, and the bulk will become overexcited in this model. We suspect that this is a quite general feature. In addition we point out that models with only two large dimensions have much stronger couplings between brane and bulk, since homogeneous excitations on the brane give rise to fields which do not fall off in the bulk. Our conclusion is that models with a KK threshold somewhat above an MeV are much more likely to be viable.

2. Fixing the moduli

It is natural to divide this discussion into two parts, according to the manner in which SUSY is broken. Note that supersymmetry is almost certainly broken on the brane at a scale comparable to a TeV. A much lower scale for SUSY breaking would bring us into conflict with direct searches. This however still leaves us the option of preserving SUSY in the bulk. The stabilization problem has a rather different character in this case. We begin with the case where SUSY is completely broken in the bulk as well as on the brane.

Our treatment of the non-supersymmetric case differs from that of [15] in the way in which we treat curvature and the cancellation of the effective cosmological constant. The latter is the cosmological constant in the effective theory below the KK energy scale and is the parameter actually measured by observation. It is the sum of a bulk term, a boundary term and a radiative correction term. In [15] it was assumed that the bulk cosmological constant was tuned to cancel the boundary cosmological constant. For large radius this implies that the dimensionless coefficient of the bulk term is extremely small, of order $(RM)^{-n}$, where n is the number of large compact dimensions. The boundary cosmological constant is of order M^4, with $M \sim 1$ TeV in the most ambitious models. In this case it is consistent to neglect the curvature terms in the effective action because the curvatures obtained by solving the Einstein equations in the presence of the bulk cosmological constant are smaller than or equal to the cosmological term itself. The resulting bulk geometry is approximately flat and the KK modes have masses of order $1/R$. The Compton wavelength of the radial dilaton is of order a mm. independent of the number of spatial dimensions. This follows from the fact that the dilaton is a bulk modulus and the overall scale of its potential is set by the boundary cosmological term.[4]

We believe that a more plausible mechanism for fine tuning the effective cosmological constant is obtained by allowing the curvature terms in the effective action to have the order of magnitude suggested by dimensional analysis. Then one can fine tune the cosmological constant by canceling the bulk cosmological term against the leading curvature term. The coefficient of the bulk term now only needs to be a factor of $(RM)^{-2}$ smaller than its naive expectation, in any number of dimensions. Of course, since we are talking about a single fine tuning in either case, the reader may feel that our choice is purely a matter of taste. We do not have strong arguments against this position, but will nonetheless present our results with the curvature terms in place.

We argue that the only plausible mechanism for stabilizing a large radius is to balance a large flux against the bulk cosmological constant, and curvature terms, as suggested in [15]. In this case the internal manifold will not be Ricci flat. Thus we find that the only case with Cavendish signals is that with two dimensions of millimeter scale. This scenario requires a huge flux, which although technically natural, does not seem to be a likely vacuum state for M theory. We also find that the natural scale for the bulk vacuum energy, and thus for the flux which stabilizes the radius, is much larger than that found in [15] in all dimensions above two.

Our treatment of the supersymmetric case is quite different from that of [15]. In particular, it is not correct that SUSY breaking in the brane induces a small bulk cosmological constant. The potential always falls to zero at infinite R in the SUSY case. We find an acceptable value for the dilaton mass only for rather large n. By raising the fundamental scale, and invoking SUSY broken at the TeV scale on the brane, we find a scenario with only modest values of the fluxes, and a radial dilaton within reach of gravitational experiments. This model has five dimensions, whose inverse size is of order 10 MeV.

2.1 Non-supersymmetric bulk

We will concentrate on a particular modulus, called the radial dilaton, R, that parameterizes the overall scale of the internal geometry. In order to study the regime of very large radial dilaton R we can use the techniques of low energy effective field

[4]This conclusion follows from equation (2.26) of [15] but was not emphasized in that paper.

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theory. The leading term in the large R expansion of the effective potential for R is the bulk cosmological constant: $\Lambda^{n+4}R^n$, where n is the number of large compact dimensions. From dimensional analysis, we expect that $\Lambda^{n+4} = aM^{n+4}$, with a a constant of order unity.[5]

Higher order terms in the expansion of the effective potential for large R can come from three sources. As noted in [15], if the internal space has non-zero Ricci curvature, one gets a term behaving as $M^{n+2}R^{n-2}$. Higher orders in the curvature and its covariant derivatives can also be important if n is large enough. The next to leading term is of order M^nR^{n-4}, and comes from terms quadratic in curvature. Even if the curvature vanishes, as on a torus, there is a Casimir energy of the massless modes. It is of the form c/R^4, with a coefficient c which is of order one and may be positive or negative. Other terms, which scale as various powers of R, can occur if the low energy effective field theory contains antisymmetric tensor gauge fields. From a flux Q of a field with rank p field strength tensor, threading a p-cycle of the internal manifold, we get an energy $\sim e^{-2}Q^2M^4(MR)^{n-2p}$. Here e is a dimensionless coupling which in principle could be large or small. In M Theory it would be a modulus. Thus, a truly large or small value of this parameter would require an explanation. We would again argue that the potential for this modulus would be unlikely to have a minimum at such extreme values.

If all dimensionless coefficients are of order one, and there is a stable minimum for the potential described above, then it occurs for $R \sim M^{-1}$ and gives a four dimensional cosmological constant of order M^4. However, we know that effective field theory generally gives a large value of the cosmological constant, so the simpleminded dimensional analysis argument may well be incorrect.

[5] The appearance of a bulk cosmological constant raises the specter of inflation in the extra dimensions. The authors of reference [15] dealt with this by insisting that the Hubble radius corresponding to the cosmological constant was always larger than the actual radius of the internal dimensions. This constraint can be understood in another way. If we consider solutions of the field equations involving time dependent moduli of the internal dimensions, then the moduli appear as scalar fields in a lower dimensional theory. In other words, we can imagine integrating out the massive Kaluza-Klein modes and obtaining an effective action for the moduli. If the potential for these fields (including the cosmological constant term) has a stable minimum then we have a static solution, with no inflation or other evolution. If we wish to treat Λ_b as a small parameter, consistency requires that the shifts in the massive modes, as we turn on Λ_b, be small. It is a simple exercise to show that if $\Lambda_b < M^{n-2}/R^2$, the contribution to the mass of the Kaluza-Klein states from the cosmological term is small, and the shifts are small ($\Lambda_b R^4 < R^2$). This question leads to precisely the constraint found by Dimopoulos et al., $\Lambda_b < M^{n+2}/R_o^2$. We do not see, however, that this constraint is truly necessary. Once $\Lambda_b > 1/R^2$, the cosmological term makes a large contribution to the mass of the low-lying Kaluza-Klein states, but there are perfectly good static solutions of the equations of motion. The naive idea that internal dimensions of scale R would lead to particles of mass $1/R$ is incorrect in this case, and the phenomenological consequences would be different. However, we will see below that the requirement that the four dimensional effective cosmological constant comes out consistent with observation, requires the bulk cosmological constant to be small enough that this issue never arises.

A conservative, or at least conventional, way to deal with this problem is to assume that the effective four dimensional cosmological constant vanishes (or takes on a tiny nonzero value consistent with observation) by some mechanism unknown in effective field theories, and pursue the consequences of this constraint on the rest of the physical problems at hand.

This viewpoint places a strong upper bound on the bulk cosmological constant. The effective four dimensional cosmological constant is

$$\Lambda_4 = \Lambda_{4+n} R^n + dM^{n+2} R^{n-2} + \Lambda_{boundary} + \text{radiative corrections}\,. \tag{2.1}$$

Here, d is a numerical constant, and the boundary cosmological constant is a term $\Lambda_{boundary}\sqrt{-g_{ind}}$, where $g^{\mu\nu}_{ind}$ is the metric induced on the wall by the metric in bulk. Naive dimensional analysis estimates it to be of order M^4, and we will assume that it is of this order in most of what follows. The authors of [15] have proposed a "brane crystallization" mechanism for stabilization in which it might be much larger. The idea is to have a large number of branes, each with tension of order M^4. We do not fully understand this scenario (in particular, we do not understand how Gauss' law can be satisfied in the presence of a vast number of branes with the same charge) and will not explore it here. SUSY on the brane must be broken near the TeV scale, in a phenomenologically viable theory, so a small boundary cosmological constant would definitely be an extra fine tuning.

With the conventional sign for the Einstein action, a manifold with positive integrated Ricci curvature, will make a negative contribution to the energy. If the bulk cosmological constant is positive, we can obtain a cancellation of the effective cosmological constant between these two terms. In order for this to occur, the dimensionless coefficient of the bulk cosmological constant must be very small if $R_0 M$ is very large (R_0 is the value of the radial dilaton at the minimum of the potential), $\Lambda_{4+n} \sim (R_o M)^{-2} M^{4+n}$. Of course, the full cancellation of the effective cosmological constant requires us to take into account many terms in the effective potential. This will involve further fine tuning of the coefficient of the bulk cosmological constant but will not change the order of magnitude estimate of its size.

We can obtain a similar cancellation with a negative integrated Ricci curvature and a negative cosmological constant. Note however that if both the integrated curvature and the bulk cosmological term contribute to the potential with the same sign,and if $n > 2$, then we can obtain a cancellation at large $R_0 M$ only when both of these terms are much smaller than indicated by dimensional analysis. We consider such a double fine tuning unacceptable and will not consider such scenarios further.

The careful reader may wonder why we have not considered a cancellation of the cosmological constant which involves a bulk term of natural order of magnitude, and some term lower order in R with a very large coefficient (we will encounter such a term in a moment). Such a cancellation is possible, but the resulting mass for the radial dilaton and KK modes is very large and most of the phenomenology expected for large extra dimensions disappears in this scenario.

One might have imagined that, having made the two largest terms in the effective action of the same order of magnitude, that we could obtain a suitable minimum just from these two terms. This is not the case. It is easily verified that the relevant variational equations are the euclidean Einstein equations with a cosmological constant and that the action (which is the energy in the non-compact dimensions) is a negative number of order the cosmological constant. Thus, the cancellation of the effective cosmological constant is incompatible with solving the equations with just these two terms.

Thus, some other term must come into the solution of the variational equations, which means that this term must have a coefficient much larger than is expected on the basis of naive dimensional analysis. There is a unique, technically natural way to obtain such a term. One can have a large p form flux, Q, wrapped around some p cycle of the internal manifold. The corresponding effective potential is

$$V(R) = M^4 \left[a(MR_0)^{-2}(MR)^n - b(MR)^{n-2} + \frac{e^{-2}Q^2}{(MR)^{2p-n}} \right], \tag{2.2}$$

where a and b are positive constants of order one.

The equations for a minimum with zero cosmological constant (up to corrections sub-leading in MR_0) may be written as :

$$e^{-2}Q^2 = (MR_0)^{2p-2}(b-a)\,, \tag{2.3}$$

and

$$(2p-2)(b-a) = 2a\,. \tag{2.4}$$

Thus we must have $b > a > 0$ and $p > 1$ in order to satisfy these equations. Since p is an integer, this means that $e^{-2}Q^2$ is always greater than $(MR_0)^2$. Remembering that $MR_0 \sim (M_P/M)^{2/n}$ we find that even for $M \sim 10$ TeV[6] and seven large dimensions,the flux is of order 10^4. Note further that eleven dimensional SUGRA does not have a two form field strength. Thus, in M Theory, we have $p = 4$ and the flux would be of order 10^{13} in seven large dimensions. We can do somewhat better in string theoretic limits in six large dimensions. Heterotic and Type I and IIA strings have bulk two form field strengths and yield suitable R_0 with fluxes of order 10^5. In this case one might hope to do better if the dimensionless coupling e^2 is small. In Type IIA theory the two form is a Ramond-Ramond field and we do not expect

[6]We choose this scale both because it is of order the phenomenological bounds we derive in the next section, and because it was claimed in [15] that one could obtain a minimum with 6 large dimensions and flux of order one when the fundamental scale is 10 TeV. Their claims were based on the assumption of a very small bulk cosmological constant.

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such a factor, but in the heterotic/Type I theories we would get an enhancement for magnetic fluxes at weak coupling. Note however that if we make this parameter very small we are introducing another stabilization problem. The string coupling is a modulus and it is difficult to understand how it is stabilized at a very small value. We believe that it is unreasonable to expect e^2 to be smaller than 10^{-2}. To obtain values even this small from string dynamics , one has to invoke the ill understood notion of Kahler stabilization [3]. We also note that in these scenarios, independent of the dimension, the radial dilaton mass is of order $1/R_0$.

A word should be said about the other moduli of the internal space, which we have been presuming fixed. In general, if $p < n$ and the flux is very large, this assumption is probably untenable. The potential for the other moduli will be overwhelmed by a huge flux which really wants to blow up only one p cycle of the manifold. On a torus one can "solve "this problem by putting flux on a complete set of cycles (e.g. put two form flux on both the 12 and 34 cycles of a four torus). It is not clear that this is possible on a general curved manifold. Thus, it may be that the case $p = n$ is the only one where the analysis we have made above really works. Other values of p might lead to fine tuning problems for the potentials of moduli other than the radial dilaton. We will continue to ignore this problem, but we consider it a further indication of the delicate balancing act which must be performed to find a theory with large dimensions.

Let us also discuss briefly the scenario of [15] in which the dimensionless coefficient in the bulk cosmological term is taken so small that the term is of the same order as the boundary cosmological constant. In this case, the overall scale of the potential is M^4, and for M of order a TeV, the radial dilaton Compton wavelength is within reach of proposed improvements of the Cavendish experiment, for any number of large compact dimensions. The cosmological constant is so small that it does not affect the masses of the KK modes.

Actually, one should be somewhat careful in the two dimensional case. The brane tension (boundary cosmological constant) is enhanced by a factor of $ln\ (MR_0)$ because long range fields do not fall off at infinity. This, via the fine tuning of the effective cosmological constant, in turn enhances the bulk cosmological term. As a consequence, the masses of both radial dilaton and KK modes are enhanced by a factor of $\sim \sqrt{ln\ (MR_0)} \sim 7$. Thus, one cannot probe the extra dimensions until one gets up to these higher energy scales. Combined with the lower bounds on the fundamental scale which we will derive in the next section, this result may be discouraging for the prospects of testing the two dimensional scenario in experiments on gravity at millimeter scales.

As we have emphasized, for $n > 2$ it seems likely that the curvature terms will be important. In their presence, the cancellation of the effective cosmological constant implies a much larger bulk energy density than in the almost flat case we have just discussed. In these curved scenarios we find a radial dilaton mass of the same order as the KK mass, which is of order its naive flat space value $1/R_0$. Thus the estimates of production cross sections and discovery criteria cited in [13] will not be substantially changed by our reevaluation of the stability of large dimension scenarios. One concludes then that effective field theory analysis shows that the only way to stabilize the system at large $R_0 M$ is to have a large integer flux, as originally proposed by [15]. Our analysis differs from theirs (for $n > 2$) by including curvature terms for the internal manifold, which dominate the effective potential. As a consequence, even a large number of large dimensions, with a 10 TeV fundamental scale still require integer fluxes of order 10^5. The hierarchy problem is to a large extent solved by hand.

In the low energy effective theory, large fluxes appear to be technically natural because they are quantized and conserved. At a more microscopic level one should investigate the possibility that there are quantum processes which can screen the flux by popping branes out of the vacuum. For M Theorists, the plausibility of this scenario becomes a question of whether vacuum states with large flux and flat external dimensions exist. We know of no SUSY vacuum states with this property. Fluxes have a tendency to appear in Chern Simons like terms and to be bounded by considerations of anomaly cancellation. However, the present subsection is devoted to non-supersymmetric vacuum states and our understanding of those is practically nil.

2.2 Supersymmetric stabilization and quintessence

If we assume that the system becomes supersymmetric in the large radius limit, then the story is quite different. In this case the bulk cosmological constant vanishes. Many of the terms in the potential which determine the value of the radius, are now *a priori* much smaller than the constant which determines the four dimensional cosmological constant.

It is also natural to impose a Ricci flatness condition on the bulk geometry, since this is the simplest way to preserve SUSY in the low energy theory. To be specific, we might imagine a IIB string or F theory compactification which preserves eight supercharges except on a threebrane (which is point-like in the compact dimensions) where various gauge fields live. The remaining four supercharges on the brane might be broken dynamically by gauge dynamics. The only unnatural thing about this idea in the context of a very low fundamental scale is the fact that we want the SUSY breaking scale to be close to the fundamental scale, so the treatment of the SUSY breaking dynamics by low energy effective field theory may be suspect. Since we are using this scenario merely to fix ideas, we will not explore this issue.

The leading terms in the effective action that cannot be set to zero by making technically natural discrete choices, now come from curvature squared terms. They are of order $M^4(MR)^{(n-4)}$. The overall coefficient of this term might be either positive or negative, but is naturally of order one. As before, the only way to achieve a technically natural stabilization at large radius is to introduce a term with large flux which is sub-leading in the large MR expansion. This means that $p > 2$. We note that in the presence of flux, the manifold is no longer Ricci flat, but the additional Ricci tensor term scales just like the flux term which produced it. Thus it does not change the argument substantially.

The details of the stabilization depend crucially on whether $n < 4$. If it is, then the fine tuning of the 4d effective cosmological constant does not require any parameter in the higher dimensional Lagrangian to take on a particularly small value. Stabilization is decoupled from the problem of the cosmological constant. To achieve it, the sign of the $(MR)^{(n-4)}$ term coming from the various quadratic curvature invariants must be negative. Furthermore, $e^2 Q^2 \sim (R_0 M)^{(2p-4)} \sim (M_P/M)^{2(2p-4)/n}$. The smallest flux is obtained for $p = 3$ and $n = 3$, and is 10^{10} when $M \sim 10$ TeV.

The mass of the radial dilaton is much smaller in these supersymmetric scenarios. For $n < 3$ the formula is

$$m \sim M \left(\frac{M}{M_P}\right)^{(4/n)} . \tag{2.5}$$

Even for a scale of 10 TeV and $n = 3$, this is of order 10^{-8} eV and the force coming from exchange of this particle should have been seen in existing experiments. Note that SUSY will imply that to leading order in the $1/(R_0 M)$ expansion, the couplings of this particle are universal.

When $n = 4$ the terms quadratic in curvature are R independent and are not useful in the stabilization program. The fine tuning of the observed cosmological constant again has no effect on the relevant terms in the potential,[7] which are now cubic curvature terms (scaling like $(MR)^{-2}$, and a large flux term with $p > 3$ (there are also terms with covariant derivatives of curvature which have the same scaling as the cubic ones). The conditions for stabilization give $e^2 Q^2 \sim (R_0 M)^{2p-6} \sim (M_P/M)^{(p-3)}$. The smallest value is obtained for $p = 4$ and equals 10^{15}. The radial dilaton mass comes out of order $M(M/M_P)^{3/2}$. Even for M of order 10 TeV this gives a Compton wavelength of order a kilometer. Thus, this scenario is ruled out by existing gravitational experiments, since there is no way to tune the radial dilaton couplings to be much smaller than that of gravity.

For $6 \geq n > 4$, the quadratic curvature terms in the effective action give rise to terms in the potential which grow with R. If we allow this term to be of its natural order of magnitude, then the whole extra dimension picture is modified. Dimensions of size R no longer give rise to KK states with masses of order $1/R$ because the large term in the action gives a large mass to all of the modes. Furthermore, the next largest term in the inverse RM expansion is the R independent boundary cosmological term. Thus, the coefficient c in the effective potential term $cM^4(RM)^{(n-4)}$ should be of order $(R_0 M)^{(4-n)}$ (where R_0 is the value of R at which

[7] This is because it can be viewed as the fine tuning of a much larger, but R independent, term in the potential.

the potential is minimized) in order to obtain the right order of magnitude for the observed cosmological constant. This is of course a fine tuning, but since it is the same fine tuning which sets the cosmological constant, we should discount it. A technically natural minimum at large R can be obtained by adding a large flux term , $Q^2M^4(RM)^{(n-2p)}$ with $p > 2$ ($p > 3$ if $n = 6$), to the action. Stabilization can be achieved if $Q \sim (R_0M)^{(2p-n)/2} \sim (M_P/M)^{(2p/n-1)}$. For $p = 3$ and $n = 5$ this is of order 10^3, the smallest value we have yet seen, when the fundamental scale is of order 10 TeV. Indeed, if the coupling of the three form gauge field strength is as weak as a standard model coupling, the amelioration of the flux bound mentioned above is likely to make the necessary integer of order 100, which is perhaps more palatable than the other examples we have found. If the three form is a Neveu-Schwarz field strength in weakly coupled IIB string theory, then a large weak coupling factor is natural. One would have to explain the stabilization of the string dilaton in this regime, however.

For this range of n the magnitude of the effective potential is of order M^4 for all n, so the radial dilaton mass is of order M^2/M_P for all scenarios in this group. Thus, if M is of order 1 TeV we have a Compton wavelength in the range of proposed improvements of short distance tests of gravity. In particular, this is true for the attractive scenario with $p = 3$ and $n = 5$.

We can make things even better by invoking SUSY on the brane. Suppose the fundamental scale M is 10 TeV, while SUSY is broken on the brane at around $M_{SUSY} \sim 1$ TeV. Then the equation for Q is replaced by $Q \sim (M_P/M)^{(2p/n-1)}\times$ $\times(M_{SUSY}/M)^2$, because the scale of the potential will be lowered to M^4_{SUSY}. If we take $p = 3$ and $n = 5$ as above, we get $Q \sim 10$. Taking into account the weak coupling factor mentioned above, this could be an integer flux of order one. Note further that the radial dilaton mass in this case would be in the range of Cavendish experiments. As far as we can see, this is the model with the fewest large dimensionless numbers in it, which still gives exciting near term phenomenology.

For $n = 7$ both the quadratic and cubic curvature terms in the effective action give terms in the potential which grow with R. In order to cancel the cosmological constant with only one fine tuned parameter, we must take the dimensionless coefficient of the quadratic term to be of order $(R_0M)^{-2}$. In order to find a stable minimum, we must add a large flux term of the form $Q^2M^4(RM)^{7-2p}$ with $p \geq 4$. The only value of p expected from eleven dimensional SUGRA is $p = 4$. This also gives the lowest value of flux, namely $Q^2 \sim (R_0M)^{(2p-6)}$ or $Q \sim (M_P/M)^{(2p-6)/7} \sim (M_P/M)^{2/7}$. For $M \sim 10$ TeV, this is larger than 10^4. Note also that there is no plausible weak coupling enhancement in this case, since 11D SUGRA has no dimensionless expansion parameter. The radial dilaton mass comes out of order $M(M/M_P)^{(6/7)}$ and it cannot be seen in Cavendish experiments.

We have seen that there are a variety of radial stabilization schemes whose plausibility depends on whether the underlying dynamics has stable vacua with large flux, and a single example where we can get a relatively low fundamental scale and a light radial dilaton with fluxes of order one. A much more exciting possibility in the case of bulk SUSY, is simply that the radius is not stabilized at all. If the leading term in the potential is positive and vanishes at large R then the system is driven to infinity. Thus, we might hope to explain a large current value for R, simply as a result of the slow migration of R to infinity during the expansion of the three dimensional part of the universe. R would be a form of quintessence [18]. Such a scenario would be a realization of Dirac's old idea that Newton's constant is small because the universe is old. Although we think that this is an attractive possibility, we will find several problems with it.[8]

We will assume that SUSY is broken on our brane, at the scale M. There is then a cosmological constant of order M^4 which must be canceled by fine tuning. We must also require that the additional, time dependent (because R itself is time dependent), potential energy of R is smaller than the conventional contributions to the energy density of the universe throughout most of cosmic history. Today it can be a finite fraction of closure density.

In order to achieve this goal, it is obviously best to have a leading term in the potential which is as high a power of R^{-1} as possible. The $1/R^4$ term comes from the Casimir energy, and its coefficient is expected to be of order one. The precise value of this coefficient depends on the nature of the coupling of the massless bulk fields to the SUSY breaking system on the wall. For the quintessence scenario, we must demand that the coefficient be positive.

The bound that the current Casimir energy density not exceed the critical density is $R_0 > 10^{30} M_P^{-1} \sim 10^{-3} cm$. Thus, in the context of a theory with $M \sim 1$ TeV, and $M_P \sim (R_0 M)^{n/2} M$ only the scenario with two large dimensions is viable. However, it is also hard to see how to avoid a term in the potential coming from quadratic curvature terms, which scales like $(RM)^{-2}$. In the presence of such a term, even the two dimensional case would seem to be ruled out. Let us agree to ignore this and explore the other features of the radial quintessence scenario.

To determine whether the bound on the radial potential was satisfied in the early history of the universe, we invoke another bound, the constraint on the time variation of Newton's constant. This is usually stated as $\dot{G_N}/G_N < 10^{-11}/yr$. This translates directly into a bound on the time variation of R, which implies that $\dot{R}/R < 10^{-11}/yr.$. Thus, during the time since nucleosynthesis RM has changed by at most a factor of order one. On the one hand, this implies that there is no problem with the energy density in R dominating the universe at previous eras. On the other hand it means that our hope to explain the large value of R as a consequence of the large age of the universe, cannot work. The initial value of R cannot be so very different than its value today. This is certainly disappointing. A cosmological explanation for the

[8] For an alternative model of quintessence in large radii models see ref. [22].

large value of RM would have to come from an inflationary era preceding the time of nucleosynthesis.[9]

It is worth pointing out that, in contrast to many models of quintessence, the current model is not seriously constrained by the cosmological variation of the fine structure constant. In this model gauge fields live on the brane and do not couple directly to R. Their coupling comes only through radiative corrections in which KK excitations are exchanged between the lines in a vacuum polarization diagram. The fine structure constant has the form

$$\frac{1}{\alpha} = \frac{1}{\alpha_0} + \mathcal{O}([RM]^{-q}) \,. \tag{2.6}$$

Thus

$$\frac{\dot{\alpha}}{\alpha} \sim \frac{\dot{R}}{R}[RM]^{-q} \tag{2.7}$$

Since we have already established that $RM > 10^{14}$ and $\dot{R}/R < 10^{-11}/yr$, from the bounds on the cosmological constant and the time variation of G_N, this is less than the strongest bound on the time variation of α from the Oklo natural nuclear reactor [31].

Another appealing feature of this picture is that the scale of the potential is not necessarily connected to the scale of supersymmetry breaking. This is in contrast to more conventional pictures [32], where these relations are tightly (and unacceptably) constrained.

There probably is a fine tuning problem of order $10^{-2,3}$ coming from the effect of the R field on astronomy. In this respect it behaves like a classic Brans-Dicke field.

None of these estimates touches on the question of whether the dynamics of the R field is really compatible with the bounds. That is, is the radial dilaton an acceptable dynamical model of quintessence. We will not attempt to answer this or any other cosmological question in the current paper.

However, it is clear that in the best of all possible scenarios, we can construct a viable model of radial dilaton quintessence only for $n = 2$ and that even in that case the large current value of the radius has to be put in as an initial condition sometime before the era of nucleosynthesis. It remains to be seen whether early universe cosmology and inflation can give us a natural explanation of this number.

To summarize: models with SUSY in the bulk can have similar stabilization mechanisms to non supersymmetric models. Often the radial dilaton is very light in the SUSY case, and the models are ruled out. There is however a SUSY model with five large dimensions and a 10 TeV fundamental scale, which requires no large or small parameters and has a radial dilaton which might be found in sub-millimeter

[9] We note a recent paper, [30], in which an attempt is made to explain the large value of RM in terms of a thermal effective potential in a pre BBN era.

gravitational experiments. The SUSY case also allows us to construct a model in which the radial dilaton is not stabilized and might act as a form of quintessence. Compared to most quintessence models, this scenario has less of a problem with the time variation of the cosmological constant. However, the appealing idea of explaining the large value of R via the cosmological time variation of this parameter runs into the observational bounds on the variation of Newton's constant. A viable model of R as quintessence might be constructed in the case $n = 2$, but the explanation of its current value would have to come from some very early inflationary era.

3. Flavor, CP and precision electroweak constraints

A traditional argument for a large fundamental scale has been the absence of certain flavor changing processes. The most dramatic of these is baryon number conservation. Unless the fundamental scale is higher than around 10^{15} GeV, proton stability must be protected by symmetries. However in many models of physics beyond the standard model such as the MSSM, it is already necessary to impose additional symmetries in order to suppress dangerous renormalizable and dimension 5 baryon and lepton violating operators. More elaborate symmetries can suppress baryon-number violating operators up to terms of very high dimension.

Of course, there are many other sorts of flavor violation which must be suppressed, and refs. [11, 23] discussed some aspects of this problem, and made an interesting proposal. They suggested that the underlying theory might possess some large flavor symmetry, and, in addition, several additional branes, far from "ours" on the scale M, but close when compared to the scale of compactification. The hierarchy of quark and lepton masses then arises because of a hierarchical separation of the branes. Explaining this hierarchy will raise many of the issues discussed above, but it certainly provides a way of parameterizing the breaking of chiral symmetries. Just as for the bulk moduli we have discussed above, fixing these "separation" moduli at extreme values is problematic. We have already argued that there cannot be any approximate supersymmetry in these pictures. In weakly coupled string theory, there is no potential between branes at the classical level. However, there are states whose mass grows with the separation of the branes. In supersymmetric theories, there is a cancellation between bosonic and fermionic modes, and perturbatively (and non-perturbatively, if there is enough supersymmetry) there is no potential. However, for non-supersymmetric theories, generically there is already a force between branes already at the one loop level. In weakly coupled string theory, at large distances, this force corresponds to the exchange of massless particles (gravitons, etc.) between the branes. Again, then, one expects stabilization only for brane separations of order the fundamental scale, if at all.

In the rest of this section, however, we will adopt the viewpoint of refs. [11, 23], and suppose that the solution of the flavor problems lies in a large separation of at least some of the branes. We will see that this still requires that the fundamental scale be of order 6 TeV, and suppression of CP violating effects requires either additional assumptions, a higher scale, or both.

In analyzing the effectiveness of this scheme in suppressing flavor changing processes, we will impose a strict notion of naturalness. In particular, one expects in string theory that operators permitted by symmetries are generated already at tree level. In addition, in accord with the arguments of ref. [33], one doesn't expect that the couplings should be weak. As a result, operators allowed by symmetries should be present at $\mathcal{O}(1)$ (one might argue that they should be larger). With this assumption, one should first examine constraints from *flavor conserving* operators, which, due to recent progress in precision electroweak physics, can be quite severe. These come from processes such as

- Direct searches for 4-fermi couplings at LEPII.
- Atomic parity violation.
- Limits from precision measurements of properties of the weak gauge bosons on dimension 6 operators.

For instance, data from the recently completed high luminosity LEPII run place a 95% C.L. limit on 4-lepton couplings such as

$$\frac{g^2}{M^2}\bar{\ell}_i\gamma^\mu\ell_i\bar{\ell}_j\gamma_\mu\ell_j\,, \tag{3.1}$$

of $M/g > 3$ TeV [34].

A comparable or slightly stronger limit on M/g can be found by considering the effects of operators such as

$$\frac{g^2}{M^2}\bar{\ell}_i\gamma^\mu\gamma_5\ell_i\bar{q}_j\gamma_\mu q_j\,. \tag{3.2}$$

on atomic parity violation [35].

Other dimension 6 operators such as

$$\frac{g^2}{M^2}(H^\dagger D^\mu H)(H^\dagger D_\mu H) \tag{3.3}$$

can affect the ρ parameter. Requiring $\delta\rho/\rho < .003$ places a constraint $M/g > 6$ TeV. Similarly strong constraints on M/g can be found by considering other precisely measured electroweak observables such as the total width, total leptonic width, and total hadronic width of the Z, which can be affected by operators such as

$$\frac{g^2}{M^2}(D^\nu Z_{\mu\nu})\bar{f}_i\gamma^\mu f_i \tag{3.4}$$

and

$$\frac{g^2}{M^2}(H^\dagger D^\mu H)\bar{f}_i\gamma_\mu f_i\,, \tag{3.5}$$

where f is any fermion.

In addition, any possible flavor symmetry involving the top quark and left handed bottom quark must be maximally broken, and one can also find constraints on M/g from processes such as

- The partial width $Z \to b + \bar{b}$, which is affected by operators such as $\bar{q}_{3L}\gamma_\mu q_{3L} H^\dagger D^\mu H$.

- $B_d \to \bar{B}_d$ mixing, which, after the effects of CKM missing are considered, is affected by $\bar{q}_{3L}\gamma_\mu q_{3L}\bar{q}_{3L}\gamma^\mu q_{3L}$.

These processes give a constraint on M/g of about 2 TeV.

We turn now to flavor violating processes in the first two generations. Clearly, without some assumption of underlying flavor symmetries, the scale M is constrained to be far larger than a few TeV. The authors of [11] made a set of assumptions which provide maximal suppression of unwanted processes. In the standard model, ignoring Yukawa couplings, there is a global $\mathrm{U}(3)^5$ symmetry. The first assumption is that a large discrete subgroup of this symmetry is in fact a good symmetry of the underlying theory. (These chiral symmetries must be discrete, in order to avoid light Nambu-Goldstone bosons on the branes. Also, in M Theory we do not expect global symmetries.) This symmetry is assumed to be broken on branes which are far from our own (in units of M^{-1}). There are some bulk fields which transform under these symmetries and which communicate this breaking to our wall. We will denote these generically by χ. The simplest way to suppress flavor changing neutral currents is to assume that the χ are in the representations necessary to give quark and lepton masses, i.e. there are various $\chi^{u,d}$ transforming as $(3,3,1)$ and $(3,1,3)$, respectively, under the discrete subgroups of $\mathrm{U}(3)^q \times \mathrm{U}(3)^u \times \mathrm{U}(3)^d$, and χ^ℓ transforming as $(3,3)$ under $\mathrm{U}(3)^\ell \times \mathrm{U}(3)^e$. In this picture, some masses are smaller than others because some of the branes are farther away than others, or some of the χ are heavier than others (since the χ expectation values on our brane are exponentially suppressed by the product of χ mass and the distance to the source of χ vev). For example, one can imagine a nearby wall responsible for the mass of the b quark, another, farther away, for the s quark, another for the electron mass, etc. The smallness of mixing angles could arise from the particular alignment of the symmetry breaking on different walls, for example (given the assumption of large discrete groups).

In such a picture, flavor violation is clearly suppressed by the discrete subgroup of $\mathrm{U}(3)^5$. The question is by how much. To analyze the amount of suppression of reasonably low dimension operators, we assume the discrete flavor symmetry is large enough so that we may proceed as if we have the full $\mathrm{U}(3)^5$ symmetry. First, it should be noted that CP conserving $\Delta s = 2$ processes are reasonably safe, provided the lightest χ fields are just those necessary to generate quark and lepton masses. Dangerous dimension six operators can arise from terms such as

$$\frac{g^2}{M^2}\bar{q}_L\chi^d d_R\bar{q}_L\chi^d d_R\,. \tag{3.6}$$

Here q refers generically to the left handed quarks, d to the right handed down-type quarks, χ^d to either the flavon field or to derivatives of the field with respect to coordinates transverse to the brane. There is no reason to expect any additional suppression of such derivatives, since the mass of χ is presumably of order M. The quark masses also actually receive contributions from an infinite number of operators involving χ and its derivatives. Thus the operator (3.7) need not be diagonal in the down quark mass basis since in general neither derivatives of χ^d nor the expectation value of $(\chi^d)^2$ can be diagonalized simultaneously with the expectation value of χ. It is however true that in models such as those suggested by refs. [11, 23], the various entries of the matrices indicated by $\chi^{u,d,\ell}$ will have the same order of magnitude as those of the corresponding Yukawa matrices. With these assumptions, in the down quark mass basis, one could find a $\Delta s = 2$ operator of order

$$\frac{g^2}{M^2}\left(\frac{m_s^2V_{cd}^2}{v^2}\right)\bar{d}_L s_R\bar{d}_L s_R\,. \tag{3.7}$$

The matrix element of the operator (3.7) is enhanced by a factor m_K^2/m_s^2 and by short distance QCD renormalization group effects. However the real part of $K\bar{K}$ mixing is adequately suppressed for M/g of 900 GeV. Similar operators, with d replaced by u, can contribute to $D\bar{D}$ mixing. However a M/g of order 1 TeV provides suppression consistent with current limits.

Other $\Delta s = 2$ operators arise from terms such as

$$\frac{g^2}{M^2}\bar{q}_L\chi\chi^\dagger q_L\bar{q}_L\chi\chi^\dagger q_L\,, \tag{3.8}$$

with various contractions of the indices. But these are suppressed by four powers of m_c or small CKM angles, and are less dangerous. $\Delta s = 1$ operators also provide weaker limits.

Consideration of CP violating operators provides, potentially, more stringent constraints. To explain the smallness of CP violation, CP must be a good symmetry of the bulk, violated spontaneously on a distant brane. Otherwise, unless the scale M is very big, CP violating operators such as

$$\frac{g^2}{M^2}f_{abc}G_a{}^{\mu}{}_{\nu}G_b{}^{\nu}{}_{\lambda}\tilde{G}_{c\mu}^{\lambda} \tag{3.9}$$

will make huge contributions to the dipole moment of the neutron d_n.

Assuming the CKM mechanism of CP violation, it is necessary that CP be violated on some of the branes responsible for quark masses. An order one CKM phase is not possible if CP is only violated on the branes responsible for the first generation masses. If CP is violated generically on the the branes which provide the quark masses, then operators such as (3.7) and also such as

$$\frac{g^2}{M^2} e F_{\mu\nu} \bar{q}_L \chi^d \sigma^{\mu\nu} d_R H \tag{3.10}$$

(where again χ may refer to a derivative of itself, not necessarily real in the same basis as the quark masses) may have complex coefficients. The former potentially gives too large an ϵ_K unless $M/g > 10$ TeV. The latter may give a too large d_n unless $M/g > 40$ TeV (assuming the contribution of the down quark dipole moment to d_n is given by naive dimensional analysis [36]). The contributions of the latter must be substantially suppressed if the scale M is anywhere near the electroweak scale. One way to suppress the contribution of the operator (3.10) is if the χ field is rather light compared with M so that contributions of its derivatives are suppressed. It is also quite possible that even if χ^d is not light, its derivatives are real and diagonal in the same basis as χ^d. Recall that we are assuming that the bulk physics preserves a large *discrete* subgroup of the flavor symmetries. Thus the bulk physics provides a potential for the matrices U(L) and U(R) which diagonalize χ^d, which is minimized for discrete values. It is thus plausible that these matrices are not spatially varying, since to do so might cost too much potential energy. There would therefore be a basis, in which the down masses were real and diagonal, and CP and strangeness were good symmetries until the effects of χ^u (and its derivatives) are considered. However it will still be expected that, e.g. $\langle \chi^u {\chi^u}^\dagger \chi^d \rangle$ will be complex in the quark mass basis and so operators such as

$$\left(\frac{g^2}{M^2}\right) e F_{\mu\nu} \bar{q}_L \chi^u {\chi^u}^\dagger \chi^d \sigma^{\mu\nu} d_R H \tag{3.11}$$

give a contribution to d_n. The contribution from the down quark is not dangerous, and even if the strange quark matrix elements are order 1 as given by naive dimensional analysis and indicated by several experiments [37], then one would have to have $M/g > 1$ TeV in order to suppress the contribution to d_n from the strange quark electric dipole moment.

Note that suppression of the contribution of the operator (3.10) still does not completely explain the small size of d_n which still could arise from the strong CP parameter $\bar{\theta}$.[10] More restricted assumptions about CP violation can ameliorate this problem as well as the constraints previously mentioned. One alternative is that there is no CP violation either in the bulk or on the branes which serve as sources

[10] This strong CP problem might still be solved by an invisible axion in the bulk, see ref. [12], or a massless up quark.

for $\chi^{u,d}$. Then the quark mass matrix is nearly real and the phase in the CKM matrix is very small.[11] This could have the advantage of solving the strong CP problem. However, one then must hypothesize more complicated mechanisms to provide CP violation in $K-\bar{K}$ mixing. For instance there could be another distant wall, on which CP and, say, the flavor symmetry acting on the left handed quarks is broken, but the other flavor symmetries are conserved. Thus this wall cannot serve as a source for $\chi^{u,d}$. The expectation value of a heavy χ^q field transforming as a 27 under the SU(3) of the left handed quarks could provide a $\Delta s = 2$ CP violating operator

$$\bar{s}_L \gamma_\mu d_L \bar{s}_L \gamma^\mu d_L \, . \tag{3.12}$$

Quark electric dipole moments would now arise only from additionally suppressed operators such as

$$\frac{g^2}{M^2} e F_{\mu\nu} \bar{q}_L \chi^q \chi^u \chi^{u\dagger} \chi^d \sigma^{\mu\nu} d_R H \, . \tag{3.13}$$

The strong CP parameter $\bar{\theta}$ and dangerous CP violating operators such as (3.9) could be sufficiently small provided that any χ fields which combine into a complex flavor singlet have enough suppression of the product of their expectation values. Within a few years such a solution will be definitely tested by the B factories, which might provide direct evidence for a nonzero CKM phase.

Lepton number and lepton flavor violation also must be highly suppressed. It is not reasonable simply to assume that the individual lepton flavors are conserved since there is good evidence for violation of lepton flavor in neutrino oscillations. One might assume that small Dirac neutrino masses arise from the mechanism of ref. [11, 39]. Neutrino masses then imply large violation of the U(3) symmetries of the left handed leptons, which we parameterize by χ^ν. Lepton flavor violation from higher dimension operators such as

$$\frac{g^2}{M^2} \bar{\ell}_L \chi^\nu \chi^{\nu\dagger} \gamma_\mu \ell_L \bar{\ell}_L \gamma^\mu \ell_L \, , \tag{3.14}$$

could lead to visible nonstandard decays such as $\tau \to 3\mu$ or $\mu \to 3e$ unless M/g is very large. However by choosing different properties for the right handed neutrinos or a different mechanism for neutrino masses one clearly has the option of assuming that lepton flavor violating χ expectation values are very small and not dangerous.[12] Because there are many possible options for the neutrino masses, we do not consider lepton flavor violating constraints further in this paper.

[11] Unlike in the standard model case [38], current data still allows a real CKM matrix if there are nonstandard contributions to $B\bar{B}$ mixing.

[12] For another discussion of neutrino masses and large extra dimensions see ref. [40].

However even with suppression of lepton flavor violation, a constraint comes from a possible contribution to the anomalous magnetic moment of the muon,[13] from

$$\frac{g^2}{M^2} e F_{\mu\nu} \bar{\ell}_L \chi^\ell \sigma_{\mu\nu} e_R H \,. \tag{3.15}$$

This is too large unless $M/g > 1$ TeV.

We have seen that consideration of the effects of flavor conserving higher dimension operators suggest that the fundamental scale should be at least 6 TeV, and in many models of CP violation the scale must be at least 10 TeV. Also a peculiar flavor symmetry is required which must be very judiciously broken. These scales are somewhat troubling from the perspective of understanding the lightness of the Higgs particle, which requires some mechanism to suppress its mass squared term to of order (200 GeV)^2–(800 GeV)^2. If this small term arises by accident, a fine tuning is required which is greater than at least a part in 100. Of course, it is possible that for some mysterious reason the natural size of the Higgs mass is not M. There might be some approximation in which the Higgs is light, and receives its mass radiatively. Also, there might be very small couplings that enter in the higher dimension operators. However avoiding substantial fine tuning of the weak scale clearly places additional nontrivial constraints on the underlying theory.

4. Cosmology

The cosmology of theories with several large dimensions could potentially be quite rich. At very early times, the gravitational and gauge couplings could be far from their present values. The conventional horizon and flatness problems might take a quite different form, and might be amenable to quite different solutions, than usually assumed.

Even so, cosmology is likely to pose serious problems for theories with such light moduli. Lacking a detailed cosmological model, we will content ourselves with a few brief remarks in this section.

First, as noted in [12], it is necessary that the bulk moduli are in their ground states to a high degree of accuracy at early times. Indeed, these authors define a "normalcy temperature", T_n, above which thermal production on the branes will over-produce the bulk modes. This temperature is quite low. In the case of two dimensions it is barely above nucleosynthesis temperatures. So it is necessary that "inflation" only reheat to temperatures very slightly above an MeV. In all cases, the temperature is orders of magnitude below the fundamental scale.

We find this condition quite puzzling. For example, if the inflaton is a bulk modulus, it will contribute to the bulk cosmological constant and inevitably displace

[13] A similar bound may also be placed by considering the explicit contributions from KK modes [41].

the radial dilaton far from its true minimum. On the other hand, without inflation in the bulk, it is hard to understand how the bulk system got into its ground state. So one must have inflation in the bulk with a reheat temperature below an MeV. Furthermore, in order to account for the excitation of the standard model during nucleosynthesis, the bulk inflation (or some other mechanism) must, as discussed in [12], leave over some excited field on the brane which dumps its energy almost entirely into standard model degrees of freedom. This seems particularly difficult in scenarios with only two large extra dimensions. In this case, homogeneous excitations on the brane have logarithmically growing couplings to bulk modes [25]. Finally, assuming that all of these other criteria have been met, the reheat temperature on the brane must be lower than the normalcy temperature of [12].

The potential difficulty posed by this last constraint is illustrated by the otherwise attractive model of inflation proposed in [29] (another model for brane inflation appears in [26]). These authors make the very interesting point that in the brane scenario there are natural candidate inflatons. These are the fields which describe the separation of the branes. These fields have potentials, arising from massless exchanges, which fall rapidly to zero when the branes are separated. When the branes are nearby, they are expected to have potentials with curvature of order M. Thus if the ground state has some branes close together, and yet they start out well separated at early times, the system can inflate. However, there are at least two difficulties with such a picture. First, as pointed out by the authors, it is difficult to have sufficiently large fluctuations. Second, the natural reheating temperature is of order M. Smaller scales seem to require fine tuning.

Finally, particularly for the case $n = 2$, there are potentially efficient production mechanisms for the bulk modes , which have not been carefully studied. Suppose for example that the universe undergoes a phase transition on a time scale small compared to R_0. This would be the case in the brane separation transition described above. Recall that for $n = 2$ there is enormous energy stored in the gravitational field surrounding the brane, spread over a millimeter. If the transition is too rapid, this energy cannot be dissipated adiabatically; it will be principally radiated in bulk modes.

We are not sure that these problems are insurmountable, and existing models of inflation also have their difficulties. Still, absent a concrete model, one is entitled to be skeptical of the possibility of arranging such a delicate sequence of events in the early universe. It is interesting that models with dimensions of order a $(10\ \mathrm{MeV})^{-1}$ or smaller avoid most of the difficulties with Big Bang Nucleosynthesis, because the KK modes are unexcited at nucleosynthesis temperatures. This is another reason why we consider this the most plausible realization of the large extra dimension scenario.

Another issue that will have to be resolved in these theories is an analog of the cosmological moduli problem. We have argued quite generally that the radial dilaton will be a field with gravitational couplings, mass of order $10^{-(3\ or\ 4)}$ eV, and a potential energy density of order a (TeV)^4. Thus, a mechanism for setting this field at its minimum before or during inflation must be found in order to avoid a matter dominated universe at nucleosynthesis energies. In scenarios with two large dimensions there will be similar problems with the KK modes.

These problems appear formidable to us, but the cosmology of brane worlds has many potential sources of surprise. The interplay of inflation on and off the brane and a rich spectrum of energy scales seems quite complicated.

5. Conclusions

The possibility that the fundamental scales of nature are comparable to the electroweak scale, while the scales of compactification are large, is extremely exciting. In this note we have argued that while this possibility is not ruled out by any phenomenological considerations, it is highly constrained, particularly within the framework of M Theory. We argued that purely phenomenological constraints, coming from precision electroweak measurements as well as flavor violating rare processes, push the lowest allowed value of the fundamental scale up to $6-10$ TeV. We believe that we have been very conservative in deriving these bounds, and used ameliorating assumptions proposed by other authors with a large degree of faith. In particular, although the authors of [11] propose a resolution of the flavor problem employing large discrete nonabelian symmetries, broken on distant branes, no explicit models with all the required properties have been constructed.[14] Our analysis assumed that models incorporating these ideas will eventually appear. If not, the flavor constraints are probably stronger. We have made similar, maximally mitigating, assumptions with respect to CP violation.

Probably the most severe problems with models of large dimensions were associated with the stabilization of the radius. In the non-supersymmetric case, it is necessary that there be large, quantized fluxes of magnitude at least 10^5. It is not clear whether M Theory allows such large fluxes in spaces which are Minkowski space times a compact manifold. When the bulk theory obeys SUSY, we saw that if there are five large dimensions, and one is willing to push the fundamental scale up to at least 10 TeV (which anyway one must do to satisfy constraints from precision experiments), while keeping the boundary cosmological constant scale fixed at 1 TeV (e.g. by invoking SUSY on the brane), then one can stabilize the radius with moderate values of the flux. The combination of experimental constraints and plausible stabilization mechanisms suggest that this kind of model is the most likely realization of large dimension scenarios. There are many other SUSY scenarios which require large values of flux and have radial dilaton masses which contradict experiment.

[14] For a slightly different and more explicit flavor proposal see ref. [42].

An interesting possibility, which we had hoped would provide more motivation for the idea of large dimensions, is that the radial dilaton might play the role of the quintessence field. In the case of two large dimensions, the scales at least are plausible: assuming a non-zero Casimir potential, the mass of the radial dilaton is naturally within a few orders of magnitude of the present value of the Hubble constant. However, one must fine tune the coefficient of a term in the effective action quadratic in curvatures (or find a model with vanishing bulk curvature) in order to have the Casimir energy dominate the potential. The cancellation of the cosmological constant between boundary and bulk effects is still mysterious, but at least the value of the radius dependent terms is of the right order of magnitude.There are two problems with this idea. The Brans-Dicke coupling of the radial dilaton is naturally of order one, whereas observation restricts its value to 10^{-4} or so. And bounds on the time variation of Newton's constant imply that one cannot explain the current large value of the radius as a consequence of the evolution of the universe during the long period of time since BBN. Still, given that the rest of the numerology is so suggestive, and also the high degree of fine tuning required by existing quintessence models [20], it is probably worth exploring this intriguing idea further. To do so, we would have to understand brane world models during inflationary eras. The analysis of such scenarios is only just beginning.

There are in fact a large number of interesting questions about large dimension scenarios that can only be understood in the context of inflationary cosmology. In particular, for those values of n for which the bulk KK spectrum is below 1 MeV, the initial conditions one must assume in order to make the model consistent with BBN are quite bizarre. The brane is excited to a temperature above 1 MeV while the bulk is in its ground state. This could only be accounted for by some inflationary mechanism which put both bulk and boundary into their ground states apart from some stretched scalar field on the brane.[15] This scalar must couple strongly to the standard model in order to dump most of its energy on the brane. On the other hand, its reheat temperature must be low (not more than a few GeV) in order to avoid excitation of the bulk through its well understood couplings to the standard model. Furthermore, baryogenesis and structure formation must all be squeezed into this rather abbreviated cosmic history. The absence of coupling to the bulk is particularly hard to understand in models with two large dimensions, where homogeneous excitations on the brane give rise to logarithmically growing effects in the bulk.

The inflationary cosmology of brane worlds is only beginning to be explored [29] and it remains to be seen whether models which meet all the challenges can be constructed. Here we note (once again) only that many of these problems are ameliorated in scenarios with a large number of dimensions with inverse size of order

[15] T.B. thanks Nima Arkani-Hamed for explaining this scenario to us.

10 MeV. The KK modes are now above nucleosynthesis temperatures. Thus, from the cosmological point of view as well, models with a large number of large dimensions and a 10 TeV fundamental scale seem like the most plausible realization of large dimension scenarios. These models could have exciting phenomenology both in gravitational and accelerator experiments and we believe they deserve further study.

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